Life-HTS

Liège University finite element models for High-Temperature Superconductors

C. Geuzaine

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Joint work with J. Dular and B. Vanderheyden

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Some background

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- We write quite a lot of codes, some released as open source software: `https://gmsh.info`, `https://getdp.info`, `https://onelab.info`

Gmsh, GetDP & ONELAB

- Gmsh (https://gmsh.info) is a 3D finite element mesh generator with a built-in CAD engine and post-processor
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- ONELAB (https://onelab.info) is an abstract interface for sharing information between codes

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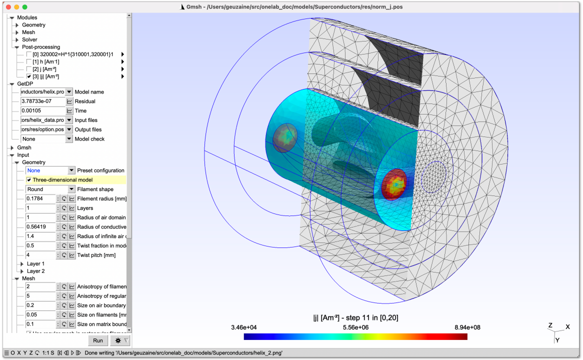
- About 20,000 downloads per month (70% Windows)
- About 800 citations per year
- Gmsh has become one of the most popular open source finite element mesh generators ($>$ 5000 citations)

Short demo

2D and 3D $\boldsymbol{h}$-ϕ GetDP formulation for twisted HTS wires, with automatic computation of cuts using the Gmsh cohomology solver [Geuzaine, EUCAS 2015]



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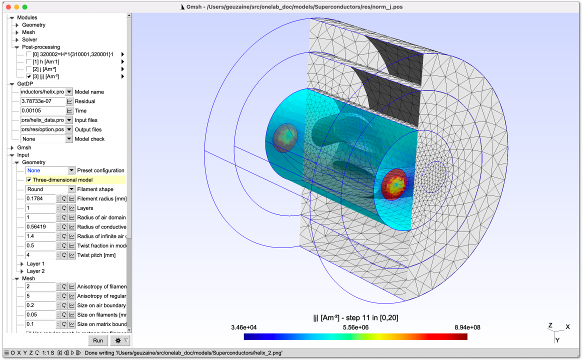

To give it a try, download the ONELAB software bundle from `https://onelab.info` and open `models/Superconductors/helix.pro`

Life-HTS

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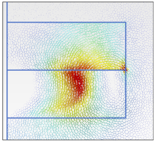

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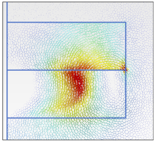

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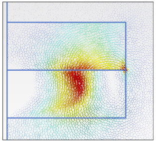

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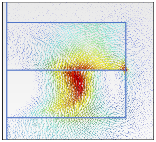

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    - Provide great flexibility in the choice of finite element formulations and associated numerical tools

Life-HTS: Why is flexibility needed?

- Life-HTS is about solving Maxwell's equations in the magnetodynamic (magneto-quasistatic) approximation

$$\textbf{curl}\, \boldsymbol{h} = \boldsymbol{j}, \quad \textbf{curl}\, \boldsymbol{e} = -\partial_t \boldsymbol{b}, \quad \operatorname{div} \boldsymbol{b} = 0,$$

  with

  - $\boldsymbol{h}$ the magnetic field (A/m),
  - $\boldsymbol{j}$ the current density (A/m^2),
  - $\boldsymbol{e}$ the electric field (V/m), and
  - $\boldsymbol{b}$ the magnetic flux density (T),

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  while the displacement current $\partial_t \boldsymbol{d}$ is neglected
- Boundary conditions and constitutive laws relating $\boldsymbol{b}$ to $\boldsymbol{h}$ and $\boldsymbol{e}$ to $\boldsymbol{j}$ are needed to obtain a well-posed problem

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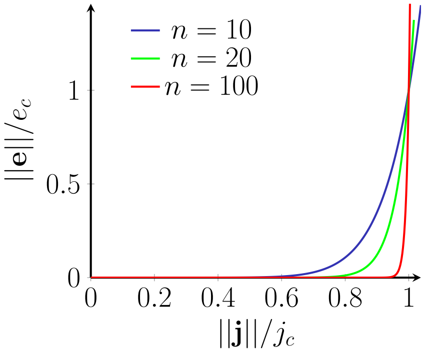
- In ferromagnetic materials: classical anhysteretic saturation law, or energy-based hysteresis model [Jacques et al., AIP Advances 2018]
- In high-temperature superconductors: $\boldsymbol{e} = \rho(\boldsymbol{j})\boldsymbol{j}$ with power law



$$\rho(\boldsymbol{j}) = \frac{e_c}{j_c}\left(\frac{||\boldsymbol{j}||}{j_c}\right)^{n-1}$$

with

- $e_c = 10^{-4}$ V/m
- j_c the critical current density
- n the flux creep exponent ($n \in [10, 1000]$)

[Plummer & Evetts, IEEE TAS 1987; Zeldov et al., Appl. Phys. Lett. 1990]

Life-HTS: Why is flexibility needed?

- Putting it all together on a Tonti diagram, along with scalar and vector magnetic and electric potentials:

$$
\begin{tikzcd}
(\phi, \omega) \arrow[r, "\textbf{grad}_h"] & \boldsymbol{h}\ (\boldsymbol{t}) \arrow[r, "\textbf{curl}_h"] \arrow[d, leftrightarrow, "\boldsymbol{b} = \mu(\boldsymbol{b})\boldsymbol{h}"] & \boldsymbol{j} \arrow[r, "\mathrm{div}_h"] \arrow[d, leftrightarrow, "\boldsymbol{e} = \rho(\boldsymbol{j})\boldsymbol{j}"] & 0 \\
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  - $\boldsymbol{b}$-conform formulations ($\boldsymbol{a}$, $\boldsymbol{a}$-v, $\boldsymbol{a}^*$, ...) satisfy the bottom exactly
- The choice of the formulation has a significant effect on the numerical performance of the finite element solver

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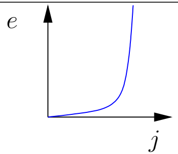
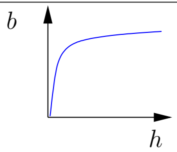
Constitutive law		
Linearization	Newton-Raphson	Picard

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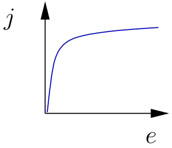
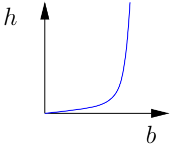
Constitutive law		
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[Dular, Geuzaine & Vanderheyden, IEEE TAS 2019]

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- Additional flexibility is required for handling HTS tapes: thin shell approximation (or not), homogenization of multiple tapes (or not), ...

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- $\boldsymbol{h}$- and $\boldsymbol{b}$-conform formulations, uncoupled or coupled ($\boldsymbol{h}$, $\boldsymbol{h}$-ϕ, $\boldsymbol{t}$-ω, $\boldsymbol{a}$, $\boldsymbol{a}$-v, $\boldsymbol{a}^*$, $\boldsymbol{h}$-(ϕ-)$\boldsymbol{a}$, $\boldsymbol{t}$-$\boldsymbol{a}$, ...):
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- Easy coupling of fields and formulations, staggered or monolithic, for multi-physics coupling (mechanical, thermal)
  - E.g. explicit Jacobian for strongly coupled magneto-thermal problem

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- Parameterizable graphical user interface through ONELAB
  - control any simulation parameter
  - construct application-specific tools for both education and industry

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- At the core of template `.pro` files, weak formulations are written symbolically

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For example, in a `.pro` file, the weak formulation: Find $u(x) \in H_0^1(\Omega)$ such that

$$-\int_\Omega a(x)\nabla u \cdot \nabla u'\, d\Omega = \int_\Omega f(x)\, u'\, d\Omega, \quad \forall u' \in H_0^1(\Omega)$$

is transcribed as

```
Formulation{
  { Name MyFirstFormulation; Type FemEquation;
    Quantity {
      { Name u; Type Local; NameOfSpace H1_0; }
    }
    Equation {
      Integral { [ -a[] * Dof{d u}, {d u} ];
        In Omega; Integration I; Jacobian J; }
      Integral { [ -f[], {u} ];
        In Omega; Integration I; Jacobian J; }
    }
  }
}
```

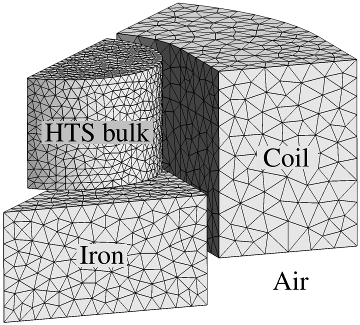
Life-HTS: Main features

Similarly, here is a bare-bones $\boldsymbol{h}$ or $\boldsymbol{h}$-ϕ formulations (they are the same—only the function space HSpace changes!)

```
Formulation {
  { Name MagDynH; Type FemEquation;
    Quantity {
      { Name h; Type Local; NameOfSpace HSpace; }
    }
    Equation {
      Integral { DtDof [ mu[] * Dof{h} , {h} ];
        In Omega; Integration Int; Jacobian Vol;  }
      Integral { [ rho[{d h}] * {d h} , {d h} ];
        In OmegaC; Integration Int; Jacobian Vol;  }
      Integral { [ dEdJ[{d h}] * Dof{d h} , {d h} ];
        In OmegaC; Integration Int; Jacobian Vol;  }
      Integral { [ - dEdJ[{d h}] * {d h} , {d h} ];
        In OmegaC ; Integration Int; Jacobian Vol;  }
    }
  }
}
```

Examples

3D HTS Magnet Motor Pole Modelling



One eight of the geometry
(air domain not shown)

Formulation	Function space	Support of DOFs	$\sigma \neq 0$ in Ω_c^C?
$\boldsymbol{h}$-formulation	$\mathcal{H}(\Omega) = \{\boldsymbol{h} \in H(\mathbf{curl};\Omega)\}$	Edges in Ω	Yes
$\boldsymbol{h}$-ϕ-formulation	$\mathcal{H}_\phi(\Omega) = \{\boldsymbol{h} \in H(\mathbf{curl};\Omega) \mid \mathbf{curl}\,\boldsymbol{h} = \mathbf{0} \text{ in } \Omega_c^C\}$	Edges in Ω_c, nodes in Ω_c^C	No
$\bar{\boldsymbol{a}}$-formulation	$\bar{\mathcal{A}}(\Omega) = \{\boldsymbol{a} \in H(\mathbf{curl};\Omega)\}$	Edges in Ω	(Yes)
$\boldsymbol{a}$-formulation	$\mathcal{A}(\Omega) = \{\boldsymbol{a} \in H(\mathbf{curl};\Omega) \mid \text{co-tree gauge in } \Omega_c^C\}$	Edges in Ω_c, facets in Ω_c^C	No
$\boldsymbol{h}$-$\boldsymbol{a}$-formulation	$\boldsymbol{h} \in \mathcal{H}(\Omega_c)$, $\boldsymbol{a} \in \mathcal{A}(\Omega_c^C)$	Edges in Ω_c, facets in Ω_c^C	No
$\boldsymbol{h}$-ϕ-$\boldsymbol{a}$-formulation	$\boldsymbol{h} \in \mathcal{H}_\phi(\Omega_a)$, $\boldsymbol{a} \in \mathcal{A}(\Omega_a)$	Edges in $\Omega_{h,c}$, nodes in $\Omega_{h,c}^C$, facets in Ω_a	No

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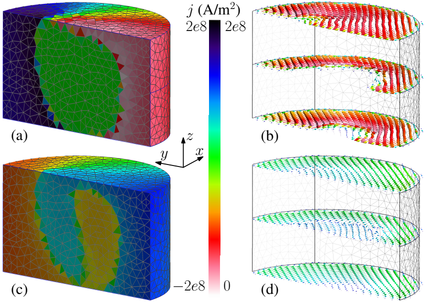


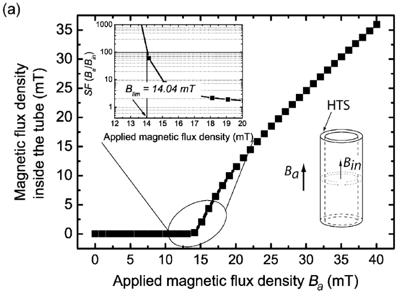
Current density in the bulk during magnetizing pulse and relaxation

[Dular et al., 2021]

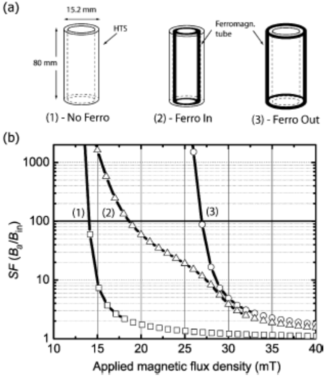
Formulation	# DOFs	# iterations	CPU time
$\boldsymbol{h}$-formulation	35,532	4,057	5h58
$\boldsymbol{h}$-ϕ-formulation	12,172	3,937	3h38
$\bar{\boldsymbol{a}}$-formulation	29,010	2,955	4h45
$\boldsymbol{a}$-formulation	26,964	3,147	3h07
$\boldsymbol{h}$-$\boldsymbol{a}$-formulation	32,045	1,124	1h25
$\boldsymbol{h}$-ϕ-$\boldsymbol{a}$-formulation	16,070	1,108	1h16

Improving HTS magnetic shields with a soft ferromagnetic material

Shielding an axial field with a HTS tube



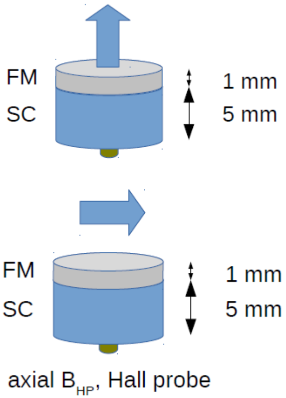
Shielding with an additional ferromagnetic tube



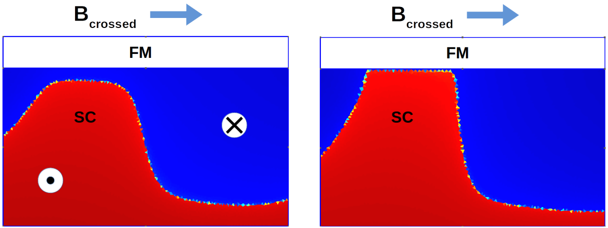
[Lousberg et al., TAS 2010]

Protecting a bulk HTS against crossed-field demagnetisation with a ferromagnetic layer

Sequence of applied fields

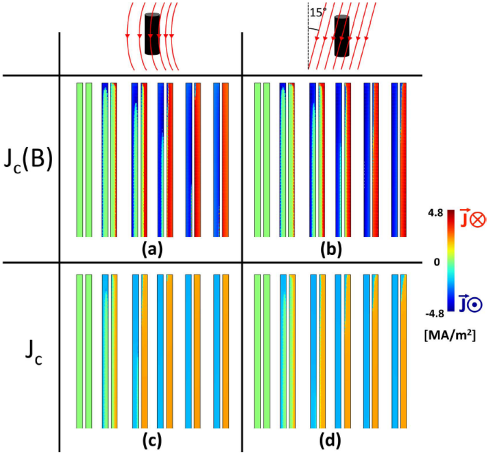


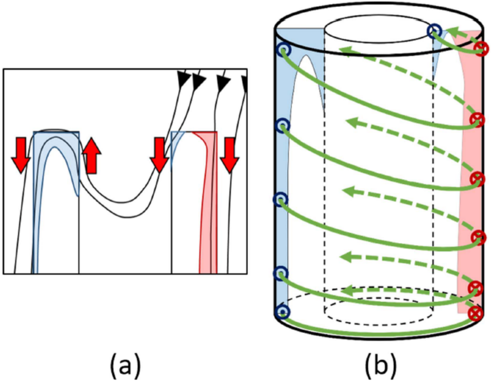
Current distribution in the bulk with a ferromagnetic top layer ($\mu_r = 10, 100$)



[Fagnard et al., SUST 2016]

Magnetic shielding in inhomogeneous fields

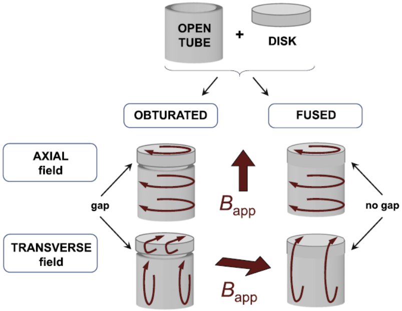




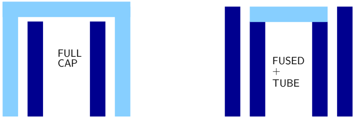
[Hogan et al., SUST 2018]

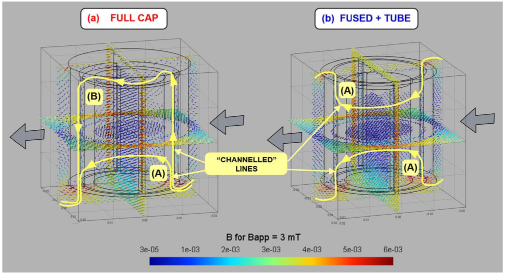
Magnetic shielding, bulk superconducting cylinders and caps

Induced currents vs. geometries



Tracking stray fields in composite shields

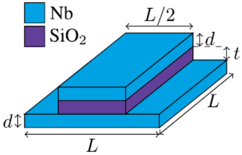




[Fagnard et al., SUST 2019]

Critical states in stacked Niobium films

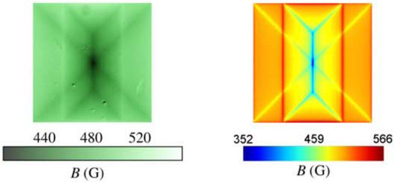
Peculiar patterns of discontinuity lines in stacks of Nb films



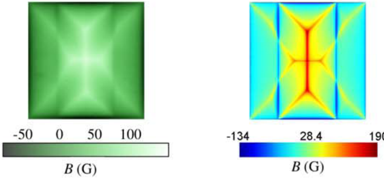
$L = 200\ \mu$m, $d = t = 300$ nm

Needs to include a genuine $J_c(B)$-dependence!

Raising field stage



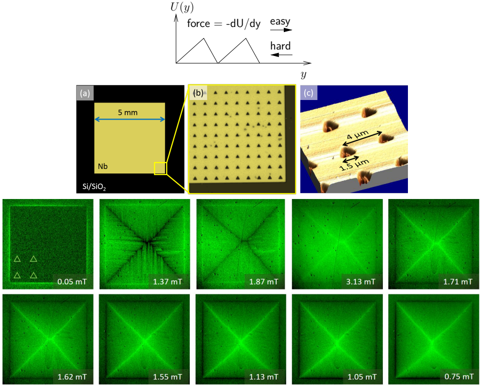
Decreasing field stage



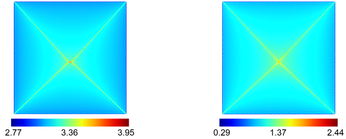
[Burger et al., SUST 2019]

Critical states in the presence of a ratchet pinning potential

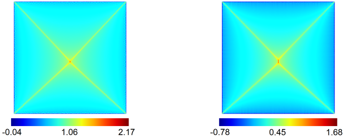
Experiment: rotation of the central discontinuity line in the decreasing field stage, after magnetization



Model: an anisotropic pinning force reproduces the result



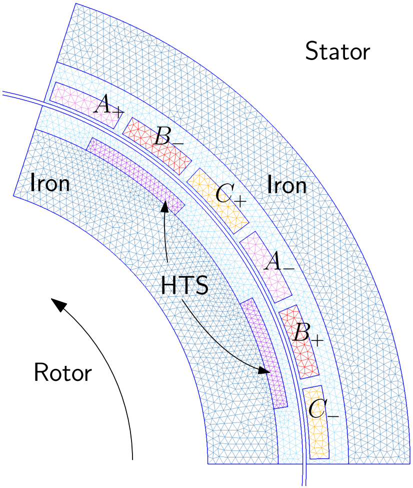
$\mu_0 H_a = 3.11$ mT $\mu_0 H_a = 1$ mT



$\mu_0 H_a = 0.75$ mT $\mu_0 H_a = 0$ mT

[Motta et al., Phys. Rev. B, in press.]

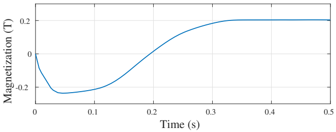
Rotating HTS motor



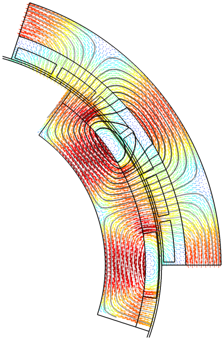
[HTS motors School 2020]

Pulse magnetization (h-a-formulation)

$$I_{B_\pm}(t) = -I_{C_\pm}(t) = \pm I_{\text{max}} \frac{t}{\tau} \exp(1 - t/\tau), I_{A_\pm}(t) = 0$$



3-phase (A-B-C) motor mode (a-formulation)



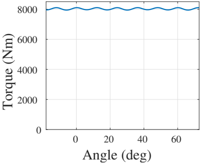
$$I_{A_\pm}(t) = \pm I_{\text{max}} \sin(\omega t)$$

$$I_{B_\pm}(t) =$$

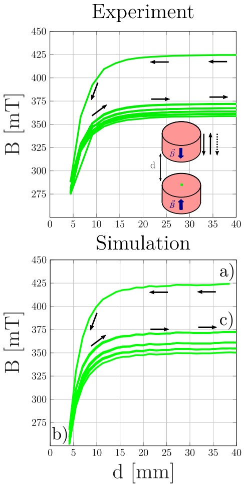
$$\pm I_{\text{max}} \sin(\omega t + 2\pi/3)$$

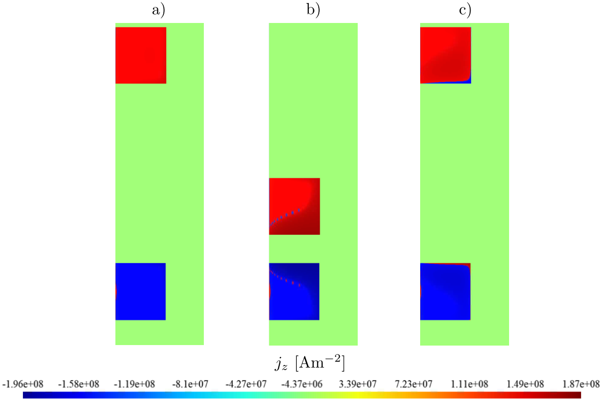
$$I_{C_\pm}(t) =$$

$$\pm I_{\text{max}} \sin(\omega t - 2\pi/3)$$



2D axisymmetric model of moving bulk superconductors

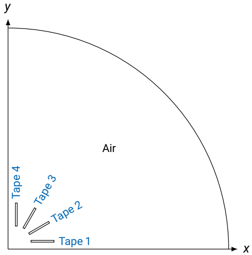


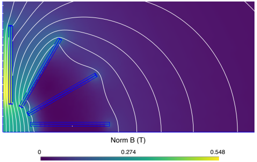


[Houbart & Vanderbemden, 2021]

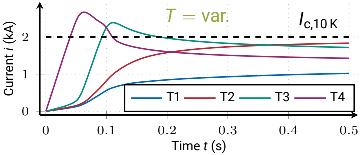
Coil of HTS Tapes

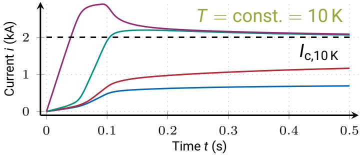
$\boldsymbol{h}$-$\boldsymbol{a}^*$ formulation with thermal coupling; tapes in parallel, series or end-coupled

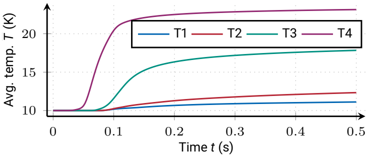


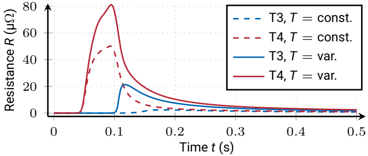


Current redistribution phenomena for current-driven tapes connected in parallel









Good agreement with reference results from COMSOL

[Schnaubelt, Bortot & Schöps, 2021]

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Thanks for your attention

http://www.life-hts.uliege.be

✉ cgeuzaine@uliege.be